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Reg. No. : ....

··ae No. : 30344 E Sub. Code : SMMA 61

(CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

For those who joined in July 2017 onwards)

Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer:

f $f(z) = z^2$, then the value of $v(x,y)$

a) $x^2 - y^2$ (b) $2xy$

c) xy (d) $x^2 + y^2$

The complex form of CR equations

a) $f_x = -if_y$ (b) $f_x = if_y$

c) $f_y = -if_x$ (d) $f_x = f_y$

3. If C is the circle with center a and radius r, then the value of $\int_C |z'(t)|\,dt$

(a) $2\pi i$ (b) $-2\pi i$

(c) $2\pi r$ (d) $-2\pi r$

4. If C is the circle $|z-2| = 5$, then $\int_C \frac{dz}{z-3} =$ ———

(a) 0 (b) $2\pi i$

(c) $-2\pi i$ (d) πi

5. $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots = ?$

(a) $\sin z$ (b) $\cos z$

(c) $\sinh z$ (d) $\cosh z$

6. The poles of $f(z) = \frac{z^2}{(z-2)(z+3)}$

(a) 2, 3 (b) –2, 3

(c) 2, –3 (d) –2, –3

$f(z) = \frac{1}{2z^2 + 5iz - 2}$, then $\operatorname{Res}\{f(z); -i/2\} = ?$

) $\frac{1}{3}$ (b) $\frac{1}{3i}$

) $-\frac{1}{3i}$ (d) $-\frac{1}{3}$

he value of $\int_{|z|=2} \tan z\, dz$

) $2\pi i$ (b) $-2\pi i$

) $4\pi i$ (d) $-4\pi i$

he fixed point of the transformation $w = \frac{1}{z - 2i}$

) 0 (b) i

) $-i$ (d) $2i$

Which one of the following is not a bilinear ransformation

) $w = z$ (b) $w = \overline{z}$

) $w = 1 + z$ (d) $w = 1 - z$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $f(z) = \operatorname{Re} z$ is nowhere differentiable.

Or

(b) If $f(z)$ and $\overline{f(z)}$ are analytic in a region D show that $f(z)$ is constant in that region.

12. (a) Prove that $\int_{-c} f(z)\,dz = -\int_{c} f(z)\,dz$.

Or

(b) Evaluate $\int_C \frac{z\,dz}{z^2 - 1}$ where C is the positively oriented circle $|z| = 2$.

13. (a) Expand $\cos z$ into a Taylor's series about the point $z = \pi/2$.

Or

(b) Find the residue of $\frac{1}{(z^2 + a^2)^2}$ at $z = ai$.

14. (a) Evaluate $\int_C \frac{dz}{2z+3}$ where C is $|z|=2$.

Or

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

15. (a) Find the bilinear transformation which maps the point $z=-1, 1, \infty$ respectively on $w=-i, -1, i$.

Or

(b) Find the fixed points of the transformation $w=\frac{1+z}{1-z}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive the CR equations in polar co-ordinates.

Or

(b) Find the analytic function $f(z)=u+iv$ given that $u-v=e^x(\cos y - \sin y)$.

17. (a) State and prove Cauchy's integral formula.

Or

(b) State and prove fundamental theorem of algebra.

18. (a) State and prove Maclaurin's series.

Or

(b) State and prove Cauchy's residue theorem.

19. (a) Evaluate $\int_0^{\infty} \frac{x \sin x}{x^2+a^2} dx$.

Or

(b) Prove that $\int_0^{\infty} \frac{\cos x}{1+x^2} dx = \frac{\pi}{2e}$.

20. (a) Find the points where the following mappings are conformal. Also find the critical points if any (i) $w=z^n$ (ii) $w=\frac{1}{2}$.

Or

(b) Prove that a bilinear transformation preserves inverse points.